Transformation matrices and Q-error indicators

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1 Transformation matrices

For a perfectly balanced line, the modal transformation matrices to relate modal and phase quantities do not change with frequency (constant transformation matrices) and can be chosen to be real (e.g. generalized Clarke, as used by EMTP).
In the general case of the untransposed line, however, the transformation matrices change with frequency. The line currents transformation matrix T_i is the matrix that diagonalizes the product $\mathbf{Y_{phase}Z_{phase}}$ where $\mathbf{Y_{phase}}$ is the shunt admittance matrix in phase quantities and $\mathbf{Z_{phase}}$ is the series impedance matrix in phase quantities. The resulting $\mathbf{Q}$ (also called $\mathbf{T_i}$) matrix, determined by the eigenanalysis routines, is complex. To standardize the results, $\mathbf{T_i}$ is normalized, using the Euclidean Norm (whereby each column j is divided by $k_j = \sqrt{\sum Q_{ij}^2}$). The voltages transformation matrix $\mathbf{T_v}$ (which diagonalizes the reverse product $\mathbf{Z_{phase}Y_{phase}}$) is not determined by the eigenanalysis routines but calculated directly from the relationship $\mathbf{T_v} = \mathbf{T}_i^{-t}$ (where the superscript means inverse transposed).

Processing of line models in time-domain the EMTP requires real transformation matrices $\mathbf{T_i}$ and $\mathbf{T_v}$. To obtain approximate $\mathbf{T_i}$ and $\mathbf{T_v}$ matrices, the columns of $\mathbf{T_i}$ (complex) can be rotated to make the imaginary parts of its elements small and then retain only the real parts.

In the case of the pi-exact (Exact-PI) model, the final form of the model is expressed in terms of self and mutual phase quantities, and there is no impediment in using exact complex transformation matrices at each frequency at which the model is produced. This model, however, is a one-frequency model, valid for steady-state solutions but not for transients simulations.

The CP model does not take into account the frequency dependence of the line parameters. The model is formulated in terms of modal quantities, with the modal parameters R, L, and C calculated exactly at only one frequency using the exact complex transformation matrix at that frequency. Since the model assumes zero modal conductances ($G_m = 0$), the columns of the transformation matrix $\mathbf{T_i}$ are rotated to satisfy this condition. As a result of this rotation, the imaginary parts of the elements of $\mathbf{T_i}$ usually become very small. Since the EMTP requires T_i to be purely real, only the real part of $\mathbf{T_i}$ (after the indicated rotation) is retained in the model data file.

The FD model takes into account the frequency dependence of the line parameters and the distributed nature of the losses (including a finite inductance G). As in the case of the CP model, however, the FD model is formulated in terms of modal quantities, and also has the constraint of requiring a real constant transformation matrix $\mathbf{T_i}$. Even though the FD model does not assume zero modal conductances, the recommended criterion to rotate $\mathbf{T_i}$ is the same as for the CP model, that is, $\mathbf{T_i}$ is rotated to satisfy the condition $\mathbf{G_{mode}} = 0$ for $\mathbf{G_{phase}} = 0$. This default rotation can be overridden with the “Rotate matrix T_i” drop down field in the output options tab.

Since G is normally very small, the results obtained with both rotation criteria are very similar. It is nonetheless believed that the default rotation gives more physically consistent results.

2 Q-error indicators

A Q-Error table is printed out by the "Line Model" module. This table gives an indication of the possible errors when using a constant real transformation matrix **Q** ($\mathbf{T_i}$) instead of the exact complex one at each frequency. A constant real $\mathbf{T_i}$ is used in the FD and in the CP models. An exact complex $\mathbf{T_i}$ at each frequency is used in the pi-exact model.

The errors shown in the Q-Error table correspond to single-frequency steady-state comparisons for unbalanced combinations of open and short circuit conditions. In these tests, all phases at the receiving end of the line are open or all phases are shorted. Unbalanced sources are connected at the sending end of the line. The values for those sources can be specified in the "Test sources for Q-error indicators" section in the output options tab. If not specified the program will use the following internal default values:

Phase	Voltage	Angle(deg)
1	1	0
2	1	0
3	1	120
4	1	0
5	1	0
6	1	120
…		

The percent errors shown in the Q-Error table for a given frequency correspond to the phase voltage or current that has the largest error.

The Q-Error table is a *qualitative guide* and does not include all possible factors. As the frequency goes higher than about 1000 Hz, the resonant peaks in the open and short circuit response curves are relatively sharp and small phase errors can result in relatively larger magnitude differences. Another factor that must be considered in these evaluations is that small open circuit currents can be in relatively large error under unbalanced conditions. To give qualitatively meaningful results, the error comparisons in the Q-Error table do not include currents or voltages smaller than 5% of the largest values.