

Constant Parameter Transmission line



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1 Theoretical background

The Constant Parameter (CP) line is classified as a frequency independent transmission line model. Its main advantage is computational speed. It is less precise than frequency dependent line and cable models, but it can be successfully used in analysis of problems with limited frequency dispersion.

The CP line parameters are calculated at a given frequency and that is why it is labeled as a frequency independent line.

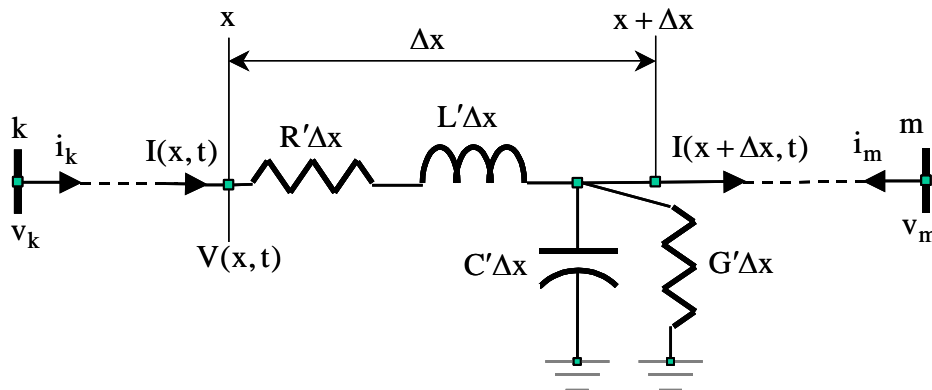


Figure 1 Distributed parameter line model

The CP line is a distributed parameter model. The basic frequency domain equations of the single phase distributed parameter line shown in Figure 1 are:

$$\frac{dV(x,t)}{dx} = -R'I(x,t) - L' \frac{dl(x,t)}{dt} \quad (1)$$

$$\frac{dl(x,t)}{dx} = -G'V(x,t) - C' \frac{dV(x,t)}{dt} \quad (2)$$

The primed variables are given in per line length. . When Laplace transformation is used:

$$\frac{dV(x,s)}{dx} = -Z' I(x,s) \quad (3)$$

$$\frac{dl(x,s)}{dx} = -Y' V(x,s) \quad (4)$$

where:

$$Z' = R' + sL' \quad (5)$$

$$Y' = G' + sC' \quad (6)$$

Differentiation of equations (3) and (4) results into:

$$\frac{d^2V(x,s)}{dx^2} = \gamma^2 V(x,s) \quad (7)$$

$$\frac{d^2l(x,s)}{dx^2} = \gamma^2 l(x,s) \quad (8)$$

with:

$$\gamma = \sqrt{(R' + sL')(G' + sC')} = \alpha + j\beta \quad (9)$$

α is the attenuation constant and β is the phase constant.

The general solution of equations (7) and (8) is given by:

$$V(x,s) = V^+ e^{-\gamma x} + V^- e^{\gamma x} \quad (10)$$

$$l(x,s) = \frac{1}{Z_c} [V^+ e^{-\gamma x} - V^- e^{\gamma x}] \quad (11)$$

with the characteristic impedance:

$$Z_c = \sqrt{\frac{R' + sL'}{G' + sC'}} = \hat{Z}_c \angle \theta_{Z_c} \quad (12)$$

Substituting equation (9) into equation (10) and assuming that V^+ and V^- are phasors ($V^+ = \hat{V}^+ \angle \theta^+$ and $V^- = \hat{V}^- \angle \theta^-$) gives:

$$V(x,s) = \hat{V}^+ e^{j\theta^+} e^{-\alpha x} e^{-j\beta x} + \hat{V}^- e^{j\theta^-} e^{\alpha x} e^{j\beta x} \quad (13)$$

The time-domain steady-state expression of this equation is:

$$V(x,t) = \hat{V}^+ e^{-\alpha x} \cos(\omega t - \beta x + \theta^+) + \hat{V}^- e^{\alpha x} \cos(\omega t + \beta x + \theta^-) \quad (14)$$

Equation (14) can be modified and reused with equation (12) to convert equation (11) into time-domain steady-state:

$$l(x,t) = \frac{\hat{V}^+}{\hat{Z}_c} e^{-\alpha x} \cos(\omega t - \beta x - \theta_{Z_c} + \theta^+) - \frac{\hat{V}^-}{\hat{Z}_c} e^{\alpha x} \cos(\omega t + \beta x - \theta_{Z_c} + \theta^-) \quad (15)$$

The term $e^{\pm \alpha x}$ is the attenuation of amplitudes of the waves. These expressions are the sums of forward traveling waves (+) and backward traveling waves (-). A generic time-domain representation is written as:

$$V(x,t) = V^+(x - vt) + V^-(x + vt) \quad (16)$$

$$l(x,t) = I^+(x - vt) + I^-(x + vt) \quad (17)$$

where wave velocity is given by:

$$v = \frac{\omega}{\beta} \quad (18)$$

The forward and backward traveling wave concept is interpreted using the illustration in Figure 2 for the waveform $V^+(x - vt)$. The traveling wave is first shown at $t = 0$ where at $x = a$ it has a value of $V^+(a)$. At any subsequent time t_x it has the same value at $x = a + vt_x$ (distortion is neglected) as it formerly had at $x = a$. It means that the voltage distribution has moved in the direction of positive x . A similar explanation is used for $V^-(x + vt)$ which is traveling in the negative x direction.

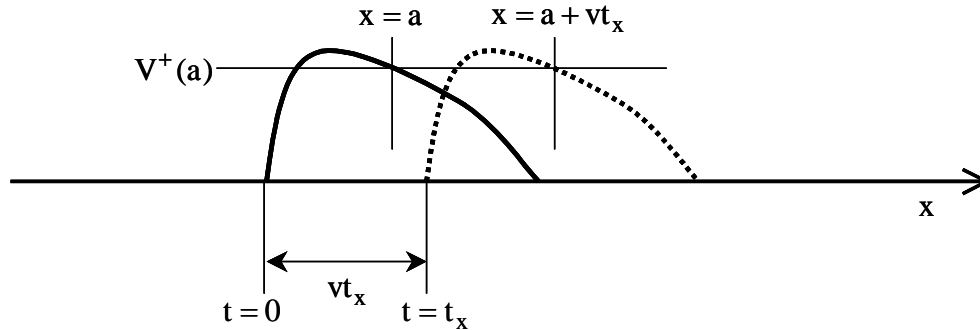


Figure 2 The traveling wave function at $t = 0$ and $t = t_x$

1.1 Steady-state model

Equations (10) and (11) are converted into the well known delay equations by applying the boundary conditions at line ends (ℓ is the line length):

$$V_k - Z_c I_k = [V_m + Z_c I_m] e^{-\gamma \ell} \quad (19)$$

$$V_k + Z_c I_k = [V_m - Z_c I_m] e^{\gamma \ell} \quad (20)$$

The frequency domain solution of the above equations results in:

$$V_k = \cosh(\gamma \ell) V_m - Z_c \sinh(\gamma \ell) I_m \quad (21)$$

$$I_k = \frac{1}{Z_c} \sinh(\gamma \ell) V_m - \cosh(\gamma \ell) I_m \quad (22)$$

These equations are often written in terms of ABCD constants

$$\begin{bmatrix} V_k \\ I_k \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_m \\ I_m \end{bmatrix} \quad (23)$$

A similar pair of equations can be written for the m side:

$$V_m = \cosh(\gamma \ell) V_k - Z_c \sinh(\gamma \ell) I_k \quad (24)$$

$$I_m = \frac{1}{Z_c} \sinh(\gamma \ell) V_k - \cosh(\gamma \ell) I_k \quad (25)$$

The corresponding frequency domain pi-circuit is shown in Figure 3 with:

$$Z_{\pi_s} = Z_c \sinh(\gamma \ell) \quad (26)$$

$$Z_{\pi_g} = \frac{Z_c}{\tanh(\gamma \ell / 2)} \quad (27)$$

The same circuit is valid for any $x = x'$ ($x' > 0$). It means that equations (24) and (25) can be also written as:

$$V(x) = \cosh(\gamma x) V_k - Z_c \sinh(\gamma x) I_k \quad (28)$$

$$I(x) = \frac{1}{Z_c} \sinh(\gamma x) V_k - \cosh(\gamma x) I_k \quad (29)$$

The circuit is derived from the nodal admittance matrix formulation:

$$\begin{bmatrix} I_k \\ I_m \end{bmatrix} = \begin{bmatrix} Y_S & -Y_M \\ -Y_M & Y_S \end{bmatrix} \begin{bmatrix} V_k \\ V_m \end{bmatrix} \quad (30)$$

with:

$$Y_S = \frac{1}{Z_c \tanh(\gamma \ell)} \quad (31)$$

$$Y_M = \frac{1}{Z_c \sinh(\gamma \ell)} \quad (32)$$

and:

$$Y_S - Y_M = \frac{1}{Z_{\pi_g}} \quad (33)$$

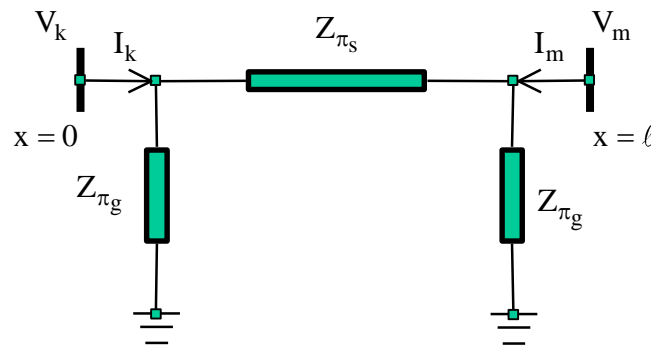


Figure 3 The frequency domain pi-circuit of a distributed parameter transmission line model

1.2 Time-domain model for CP-line

To avoid convolution problems, the CP-line model neglects the frequency dependence of parameters and first assumes a lossless line. The losses are included at a later stage.

1.2.1 Lossless line model

When the frequency dependence of line parameters is neglected and the line is assumed to be lossless, then:

$$\gamma = s\sqrt{L'C'} = j\beta \quad (34)$$

$$Z_c = \sqrt{\frac{L'}{C'}} \quad (35)$$

Z_c is now purely resistive and it is also called the surge impedance Z_s . The propagation speed is now defined by:

$$v = \frac{1}{\sqrt{L'C'}} \quad (36)$$

Equations (28) and (29) simplify to:

$$V(x) = \cos(\beta x) V_k - jZ_c \sin(\beta x) I_k \quad (37)$$

$$I(x) = j\frac{1}{Z_c} \sin(\beta x) V_k - \cos(\beta x) I_k \quad (38)$$

In which case equations (26) and (27) are now given by:

$$Z_{\pi_s} = jZ_c \sin\left(\frac{\ell}{\lambda} 2\pi\right) \quad (39)$$

$$Z_{\pi_g} = -j \frac{Z_c}{\tan\left(\frac{\ell}{\lambda} \pi\right)} \quad (40)$$

λ is the wavelength (the length corresponding to one full cycle of voltage and current):

$$\lambda = \frac{1}{f\sqrt{L'C'}} = \frac{2\pi}{\beta} \quad (41)$$

Equations (19) and (20) must be transformed back into a time-domain solution to give the delay equations:

$$v_k(t) - Z_c i_k(t) = v_m(t - \tau) + Z_c i_m(t - \tau) \quad (42)$$

$$v_m(t) - Z_c i_m(t) = v_k(t - \tau) + Z_c i_k(t - \tau) \quad (43)$$

where the propagation delay is introduced as:

$$\tau = \ell\sqrt{L'C'} \quad (44)$$

The EMTP lossless distributed parameter single phase line model is now assembled in Figure 4 from equations (42) and (43). The history terms shown in Figure 4 are given by:

$$i_{k_h} = \frac{v_m(t - \tau)}{Z_c} + i_m(t - \tau) \quad (45)$$

$$i_{m_h} = \frac{v_k(t - \tau)}{Z_c} + i_k(t - \tau) \quad (46)$$

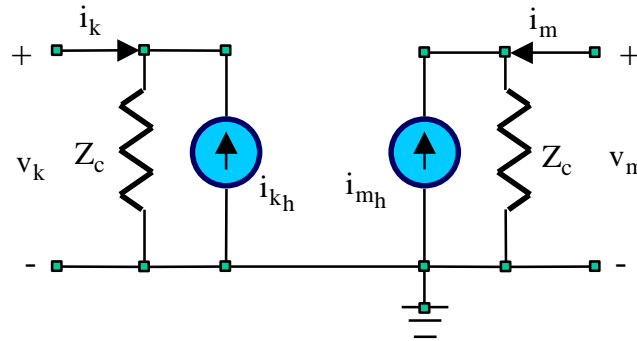


Figure 4 EMTP distributed parameter lossless line model for time-domain simulations

It is noticed that the above line model has a time-domain decoupling effect on the interconnected networks. An exact solution is found when $\tau = k\Delta t$ (k is an integer) and linear interpolation is used when $\tau \neq k\Delta t$.

1.2.2 Inclusion of losses

The losses are modeled by separating the line into two separate lossless lines of halved equal propagation time and distributing the total line resistance as shown in Figure 5 (see also [1]).



Figure 5 Inclusion of losses in the distributed parameter line model

This approach remains acceptable if $R \ll Z_c$. This condition is satisfied for most transmission lines, it may however become invalid for some cable cases.

1.2.3 Distortionless model

According to equation (9):

$$\gamma = \left[\frac{R'}{sL'} + 1 \right]^{\frac{1}{2}} \left[\frac{G'}{sC'} + 1 \right]^{\frac{1}{2}} (\sqrt{L'C'})s \quad (47)$$

It can be approximated by:

$$\gamma \approx \left[\frac{R'}{s2L'} + 1 \right] \left[\frac{G'}{s2C'} + 1 \right] (\sqrt{L'C'})s \quad (48)$$

and further simplified to:

$$\gamma \approx s\sqrt{L'C'} + Z_c \frac{G'}{2} + \frac{1}{Z_c} \frac{R'}{2} + \frac{R'G'}{4s\sqrt{L'C'}} \quad (49)$$

If G is neglected then:

$$\gamma^\ell = s\tau + \alpha\ell \quad (50)$$

with:

$$\alpha = \frac{R}{2} \frac{1}{Z_c} \quad (51)$$

It implies that equations (42) and (43) must be rewritten as:

$$v_k(t) - Z_c i_k(t) = [v_m(t - \tau) + Z_c i_m(t - \tau)] e^{-\alpha\ell} \quad (52)$$

$$v_m(t) - Z_c i_m(t) = [v_k(t - \tau) + Z_c i_k(t - \tau)] e^{-\alpha\ell} \quad (53)$$

Which makes the distortionless model of the CP-line.

1.2.4 Multiphase version

The above equations have been presented for the single phase case. For the multiphase case the basic equations (7) and (8) are rewritten in matrix form:

$$\frac{d^2 \mathbf{V}}{dx^2} = \mathbf{Z}' \mathbf{Y}' \mathbf{V} \quad (54)$$

$$\frac{d^2 \mathbf{I}}{dx^2} = \mathbf{Y}' \mathbf{Z}' \mathbf{I} \quad (55)$$

With eigenvalue theory, it becomes possible to transform the above two coupled equations from phase quantities to modal decoupled quantities. Hatted vectors are modal quantities:

$$\mathbf{V} = \mathbf{T}_v \hat{\mathbf{V}} \quad (56)$$

$$\mathbf{I} = \mathbf{T}_i \hat{\mathbf{I}} \quad (57)$$

Equations (54) and (55) are now rewritten as:

$$\frac{d^2 \mathbf{V}}{dx^2} = \mathbf{T}_v^{-1} \mathbf{Z}' \mathbf{Y}' \mathbf{T}_v \hat{\mathbf{V}} \quad (58)$$

$$\frac{d^2 \mathbf{I}}{dx^2} = \mathbf{T}_i^{-1} \mathbf{Y}' \mathbf{Z}' \mathbf{T}_i \hat{\mathbf{I}} \quad (59)$$

Multiplying by transformation matrices creates a diagonal matrix:

$$\mathbf{\Lambda} = \mathbf{T}_v^{-1} \mathbf{Z}' \mathbf{Y}' \mathbf{T}_v \quad (60)$$

It is recalled that contrary to eigenvalues, the eigenvectors are not uniquely defined. While the matrix products $\mathbf{Z}' \mathbf{Y}'$ and $\mathbf{Y}' \mathbf{Z}'$ have identical eigenvalues, their eigenvectors are different but related to each other through:

$$\mathbf{T}_i = \left(\mathbf{T}_v^t \right)^{-1} \quad (61)$$

Equations (3) and (4) are also rewritten to become multiphase and transformed into modal domain:

$$-\frac{d\hat{\mathbf{V}}}{dx} = \mathbf{T}_v^{-1} \mathbf{Z}' \mathbf{T}_i \hat{\mathbf{i}} \quad (62)$$

$$-\frac{d\hat{\mathbf{i}}}{dx} = \mathbf{T}_i^{-1} \mathbf{Y}' \mathbf{T}_v \hat{\mathbf{V}} \quad (63)$$

It appears that:

$$\hat{\mathbf{Z}} = \mathbf{T}_i^{-1} \mathbf{Z}' \mathbf{T}_i \quad (64)$$

$$\hat{\mathbf{Y}} = \mathbf{T}_v^{-1} \mathbf{Y}' \mathbf{T}_v \quad (65)$$

A completely decoupled set of equations is available at this stage. The multiphase line is transformed into a decoupled set of modal circuits. Any given mode i has its own propagation constant $\hat{\gamma}_i$ (found from equation (60)) and characteristic impedance:

$$\hat{Z}_{c_i} = \frac{\sqrt{\hat{Z}_{ii}}}{\sqrt{\hat{Y}_{ii}}} \quad (66)$$

For an untransposed line there are as many distinct modes as phases. It is noticed that CP-line uses a constant real transformation matrix, that is a transformation matrix calculated at a given model frequency.

For a completely transposed (balanced) line both \mathbf{Z}' and \mathbf{Y}' have equal diagonal (self) elements and equal off-diagonal elements. It is then possible to use a unique (\mathbf{T}_i and \mathbf{T}_v become identical) and pre-programmed transformation matrix. Only two distinct modes exist in a continuously transposed line. If S stands for self and M for mutual, then modal and phase series impedances are simply related through:

$$\hat{Z}_0 = Z'_S + (n-1)Z'_M \quad \text{for } i = 1 \quad (67)$$

$$\hat{Z}_1 = Z'_S - Z'_M \quad \text{for } i > 1 \quad (68)$$

where the two distinct modes are labeled as 0 and 1, i is the mode counter and n is the number of phases. The same treatment is applicable to \hat{Y}_0 and \hat{Y}_1 .

EMTP uses a built-in Clarke ($\alpha\beta 0$) transformation matrix for an m -phase balanced line:

$$\mathbf{T}_i = \begin{bmatrix} \frac{1}{\sqrt{m}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \cdots & \frac{1}{\sqrt{j(j-1)}} & \cdots & \frac{1}{\sqrt{m(m-1)}} \\ \frac{1}{\sqrt{m}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \cdots & \frac{1}{\sqrt{j(j-1)}} & \cdots & \frac{1}{\sqrt{m(m-1)}} \\ \frac{1}{\sqrt{m}} & 0 & \frac{-2}{\sqrt{6}} & \cdots & \cdot & \cdots & \cdot \\ \cdot & \cdot & 0 & \cdots & \frac{-(j-1)}{\sqrt{j(j-1)}} & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & 0 & \cdots & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdots & \cdot \\ \frac{1}{\sqrt{m}} & 0 & 0 & \cdots & 0 & \cdots & \frac{-(m-1)}{\sqrt{m(m-1)}} \end{bmatrix} \quad (69)$$

The case of a double-circuit line requires a different transformation matrix. A double-circuit line is made of two three-phase lines. It can be assumed to be almost balanced and given by the impedance matrix:

$$\mathbf{Z}_d = \begin{bmatrix} Z_S & Z_M & Z_M & Z_P & Z_P & Z_P \\ Z_M & Z_S & Z_M & Z_P & Z_P & Z_P \\ Z_M & Z_M & Z_S & Z_P & Z_P & Z_P \\ Z_P & Z_P & Z_P & Z_S & Z_M & Z_M \\ Z_P & Z_P & Z_P & Z_M & Z_S & Z_M \\ Z_P & Z_P & Z_P & Z_M & Z_M & Z_S \end{bmatrix} \quad (70)$$

The diagonalized version of this matrix is :

$$\hat{\mathbf{Z}}_d = \begin{bmatrix} Z_G & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{IL} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_L & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_L & 0 & 0 \\ 0 & 0 & 0 & 0 & Z_L & 0 \\ 0 & 0 & 0 & 0 & 0 & Z_L \end{bmatrix} \quad (71)$$

where:

$$Z_G = Z_S + 2Z_M + 3Z_P = Z_0 + Z_{0C} \quad (72)$$

$$Z_{IL} = Z_S + 2Z_M - 3Z_P = Z_0 - Z_{0C} \quad (73)$$

$$Z_L = Z_S - Z_M = Z_+ \quad (74)$$

Z_{0C} is the zero sequence coupling mode between the two lines. Backward transformation is given by:

$$Z_M = \frac{Z_G + Z_{IL} - 2Z_L}{6} \quad (75)$$

$$Z_S = \frac{Z_G + Z_{IL} + 4Z_L}{6} \quad (76)$$

$$Z_P = \frac{Z_G - Z_{IL}}{6} \quad (77)$$

2 Available versions

The generic version of this device is given by "CP m-phase". The other versions available in the library line.cif are:

- CP 3-phase: only for a 3-phase line.
- CP double: double-circuit transmission line.

Although "CP m-phase" can be used in all cases, the above versions are provided for convenience and simplicity of usage.

2.1 When changing phases

It is not allowed to change phases on any version of this device. Some options in the Drawing tab of this device can result in broken connections that the user might need to repair manually. It is assumed that such options are selected before making final connections to the line.

3 Parameters

The parameters given here are for the generic "CP m-phase" version.

The **Number of phases** is used to enter the number of phases. Which is also called the number of wires, since a transmission line can have ground wire in addition to phase wires. There could be several coupled wires.

The next section is used to select the type of model and the type of entered data.

The **Distortionless** checkbox is for selecting the distortionless model explained in the above theoretical section.

If the line is **Continuously transposed** then it has only two distinct propagation modes and uses a built-in transformation matrix T_i . When the line is not continuously transposed, then there are as many modes as wires and the user needs to enter a real transformation T_i .

In the **type of data**, the user can choose the input parameters for the **Propagation mode data**. The line length units can be any units since R' , L' and C' are expressed per unit length. Data is required for each mode. The parameters Z_s (characteristic impedance, also written as Z_c), v (speed) and propagation delay τ are explained in the above theoretical section: see expression (44) for τ , see expression (35) or (66) for Z_s and see equation (36) for velocity v .

The mode count starts at 1. When the line is 3-phase, the 1-mode is the same as 0-sequence, mode 2 is the same as positive sequence. When the line is balanced (continuously transposed), the mode 3 is the same as mode 2, that is positive and negative sequence data fields are identical. Modal domain is generic and used for entering data when the line is not continuously transposed.

The required data can be obtained from the Line Data device. An example is given in Line_Data\LTC5A.ecf under the EMTPWorks directory Examples. More information can be found from the tooltip of the **Load data from file** button.

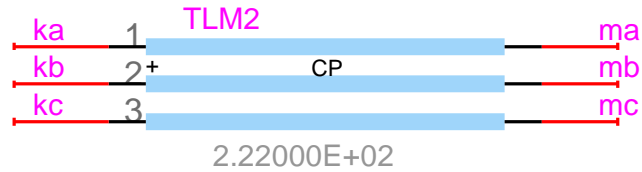
3.1 Generic rules

Although the data forms include testing for illegal data, it is not possible to test and reject all possible errors in data, specially when named values (parameters) are used instead of numbers in data fields. It is allowed to enter 0 for R' , but it is not allowed to enter 0 for L' and C' of any mode. The mode length must be also greater than 0.

It is not allowed to delete any pins or change pin attributes. The drawing of "CP m-phase" is continuously updated according to the selected number of wires and selected options in the "Drawing" tab.

4 Netlist format

The Netlist format described in this section is only valid for the “CP m-phase” device. The other versions of this line model may have some variations in their Netlist format.



This example is given for the above 3-phase line:

```

_TLM;TLM2;6;6;ka,kb,kc,ma,mb,mc,
3,0,0,3,1,1,1,1,
2.22000E+02 1.87358E-01 6.91747E+02 1.15554E-03
2.22000E+02 1.74117E-02 2.76508E+02 7.59267E-04
2.22000E+02 1.74187E-02 2.90919E+02 7.54592E-04
0.58094651 -0.40960336 0.70710678
0.57008227 0.81513620 0.00000000
0.58094651 -0.40960336 -0.70710678
0
0
0

```

Field	Description
<u>TLM</u>	Part name
TLM2	Instance name, any name.
6	Total number of pins
6	Number of pins given in this data section
ka,kb,kc	Signal names connected to k-side of this device, as many names as wires
ma,mb,mc	Signal name connected to m-pin, any name
nphase	Number of phase
0	Distortionless model when 1
0	Continuously transposed when 1
Input type	Select type of data: 1, 2 or 3, same order as in the Data tab.
Units for IC	Voltage units for initial conditions from the IC tab.
Units for parameters	Units according to Input type, same order.
Propagation data	Data for all modes, row-by-row, as many rows as wires.
Transformation matrix.	Transformation matrix.
Initial conditions	Initial condition voltages, as many rows as wires.

Comma separated data is saved into the ParamsA attributes. The rest of the data is saved into the ModelData attribute.

5 Steady-state model

Since the steady-state model is used for the initialization of the subsequent time-domain solution, EMTP uses the model of Figure 5 after replacing the lossless sections by the circuit of Figure 3 for the lossless approximation.

6 Initial conditions

The steady-state solution explained above is used to initialize the transmission line for quickly achieving a steady-state condition in time-domain at simulation startup.

The user can enter manual initial conditions for each wire. This represents trapped charge on the line. The line is also automatically initialized from the harmonic steady-state solution. The distortionless model is also correctly initialized.

If the line is energized in steady-state then the user should not also provide trapped charge data.

7 Frequency Scan model

The pi-circuit shown in Figure 3 is used at each frequency of the frequency scan. It is noticed that this is not the same as in the steady-state where the lossless version was used. It is also notice that even though the pi-section is recalculated at each frequency using equations (26) and (27) the actual variation of parameters with frequency is not accounted for.

8 Time-domain model

The time-domain model is the one shown in Figure 4. EMTP keeps and rotates a history buffer for calculating the history current sources. The buffer length depends on the propagation delay and the integration time-step. The propagation delay for each mode must be greater or equal to the integration time-step, an error message will result otherwise.

9 References

- [1] H. W. Dommel: "Digital computer solution of Electromagnetic Transients in single and multiphase networks", IEEE Transactions, Vol. PAS-88, pages 388-399, April 1969.